

<<NRICH4973>>

A decorator can buy pink paint from two manufacturers.

- Paint A is made up from red and white paint in the ratio $1:3$.
- Paint B is made up from red and white paint in the ratio $1:7$.

He can mix the paints to produce a different shade of pink.

If Paint A and Paint B come in same size cans, what is the least number he would need of each type in order to produce pink paint containing red and white in the following ratios:

- $1:4$
- $1:5$
- $1:6$

The numbers in the ratio $1:3$ of Paint A and $1:7$ of Paint B is just the ratio of red and white not the actual amount. So, to find the actual amount we need to multiply x at the ratio of red and white of Paint A.

$$\text{Total amount of the Paint A} = 1x + 3x$$

To find the actual amount of Paint B we need to multiply y at the ratio of red and white of Paint B.

$$\text{Total amount of the Paint B} = 1y + 7y$$

Because the size of the cans (A and B) are same,

$$1x + 3x = 1y + 7y$$

$$4x = 8y$$

$$\therefore x : y = 2 : 1$$

Ratio of Paint A times 2 equals ratio of Paint B (the units).

Paint A $\rightarrow R : W = 2 : 6$ } This number is the smallest number
 Paint B $\rightarrow R : W = 1 : 7$ } that makes the same units.

To make $R : W = 1 : 4$, we need to multiply a to the Paint A and b to the Paint B.

$$\text{Paint A} \rightarrow R : W = 2a : 6a$$

$$\text{Paint B} \rightarrow R : W = 1b : 7b$$

$$\begin{aligned} \text{Red} : 2a + 1b &= 1c \\ \text{White} : 6a + 7b &= 4c \end{aligned} \rightarrow \text{Red} : \text{White} = 1 : 4 \text{ in final mix.}$$

$$\begin{array}{r} 6a + 7b = 4c \\ - 8a + 4b = 4c \leftarrow (2a + 1b = 1c) \times 4 \\ \hline -2a + 3b = 0 \end{array}$$

$$3b = 2a$$

$$a : b = 3 : 2$$

$$\begin{array}{l} \therefore \text{Paint A} = 3 \text{ cans.} \\ \text{Paint B} = 2 \text{ cans.} \end{array}$$



To make $R : W = 1 : 5$

$$\begin{aligned} 2a + 1b &= 1c \\ 6a + 7b &= 5c \end{aligned} \rightarrow \text{find mix.}$$

$$\begin{array}{r} 6a + 7b = 5c \\ - 10a + 5b = 5c \\ \hline -4a + 2b = 0 \end{array}$$

$$2b = 4a$$

$$a : b = 1 : 2$$

$$\begin{array}{l} \therefore \text{Paint A} = 1 \text{ can} \\ \text{Paint B} = 2 \text{ cans.} \end{array}$$



To make $R:W = 1:6$

$$2a + 1b = 1c$$

$$6a + 7b = 6c$$

$$6a + 7b = 6c$$

$$\begin{array}{r} -) 12a + 6b = 6c \\ - 6a + 1b = 0 \end{array}$$

$$b = 6a$$

$$a:b = 1:6$$

$\therefore$ Paint A = 1 can Paint B = 6 cans
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Excellent

Another decorator buys pink paint from two different manufacturers:

- Paint C is made up from red and white paint in ratio $1:4$.
- Paint D is made up from red and white paint in ratio $1:9$.

What is the least number he would need of each type in order to produce pink paint containing red and white in the following ratios:

- $1:5$
- $1:6$
- $1:7$
- $1:8$

Ratio of Paint C times 2 equals ratio of Paint D (units).

$$\text{Paint C} \rightarrow R:W = 1:4 = \overset{R}{2}:\overset{W}{8}$$

$$\text{Paint D} \rightarrow R:W = \underset{R}{1}:\underset{W}{9}$$

$\langle 1 : 5 \rangle$

$$2c + 1d = 1e$$

$$8c + 9d = 5e$$

$$8c + 9d = 5e$$

$$-) 10c + 5d = 5e$$

$$- 2c + 4d = 0$$

$$4d = 2c$$

$$c : d = 2 : 1$$

$\therefore$ Paint C = 2 cans
Paint D = 1 can

 $\langle 1 : 6 \rangle$

$$2c + 1d = 1e$$

$$8c + 9d = 6e$$

$$8c + 9d = 6e$$

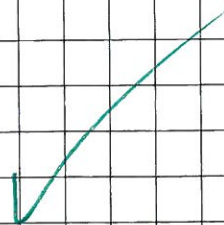
$$-) 12c + 6d = 6e$$

$$- 4c + 3d = 0$$

$$3d = 4c$$

$$c : d = 3 : 4$$

$\therefore$ Paint C = 3 cans
Paint D = 4 cans



$\langle 1 : 7 \rangle$

$$2c + 1d = 1e$$

$$8c + 9d = 7e$$

$$8c + 9d = 7e$$

$$-7 \quad 14c + 7d = 7e$$

$$-6c + 2d = 0$$

$$2d = 6c$$

$$c : d = 1 : 3$$

$$\therefore \text{Paint C} = 1 \text{ can}$$

$$\text{Paint D} = 3 \text{ cans}$$

 $\langle 1 : 8 \rangle$

$$2c + 1d = 1e$$

$$8c + 9d = 8e$$

$$8c + 9d = 8e$$

$$-16c + 8d = 8e$$

$$-2c + 1d = 0$$

$$d = 2c$$

$$c : d = 1 : 2$$

$$\therefore \text{Paint C} = 1 \text{ can}$$

$$\text{Paint D} = 2 \text{ cans}$$

